15A

1(a) $u_n = 5n$

$u_1 = 5(1) = 5$

$u_2 = 5(2) = 10$

$u_3 = 5(3) = 15$

$\vdots$

$u_{30} = 5(30) = 150$

(f) $u_n = 3^{n-1}$

$u_1 = 3^0 = 1$

$u_2 = 3^1 = 3$

$u_3 = 3^2 = 9$

$u_{30} = 3^{29} = 6.863 \times 10^{13}$ (4 S.F.)

15B

1 (a) 4, 8, 12, 16, 20, ... (+4 to previous term)

$u_{n+1} = u_n + 4$, $u_1 = 1$

(b) 6, 11, 16, ... (+5, +5)

$u_{n+1} = u_n + 5$, $u_1 = 6$

(c) 7, 10, 13, 16, ... (+3, +3)

$u_{n+1} = u_n + 3$ $u_1 = 7$

(d)

(e) 17, 13, 9, 5, ... (−4, −4, −4)

$u_{n+1} = u_n - 4$ $u_1 = 17$

(f) 2, 4, 8, 16 (×2, ×2, ×2)

$u_{n+1} = 2u_n$ $u_1 = 2$

(g) 80, −40, 20, −10 (×−½, ×−½, ×−½)

$u_{n+1} = -\frac{1}{2}u_n$ $u_1 = 80$

(h) 1, 2, 5, 14, 41 (×3−1, ×3−1, ×3−1, ×3−1)

1, 3, 9, 27

$u_{n+1} = 3u_n - 1$ $u_1 = 1$

15B

(2) amount doubles each day

$u_{n+1} = 2u_n$ $\quad u_0 = 150$, (at time=0 there are 150 termites)

(3) 18% drop so 82% = 0.82 left

$u_{n+1} = 0.82u_n$ $\quad u_0 = 40ml$

(4)

$u_{n+1} = 1.005u_n + £50$ $\quad u_0 = £100$ (initial amount)

Note: a 0.5% increase $= 100\% + 0.5\%$

$= 100.5\%$

$= 1.005$

(5) 15% drop so 85% left

$u_{n+1} = 0.85u_n + 40$ $\quad u_0 = 500ml$

15C

(1) $u_{n+1} = 2u_n - 7 \qquad u_1 = 9$

$u_2 = 2u_1 - 7$

$u_2 = 2(9) - 7$

$u_2 = 11$

$u_3 = 2u_2 - 7$

$u_3 = 2(11) - 7$

$u_3 = 15$

$u_4 = 2u_3 - 7$

$u_4 = 2(15) - 7$

$u_4 = 23$

(3) 20% reduction so 80% left

$u_{n+1} = 0.8u_n + 40 \qquad u_0 = 80$

$u_1 = 0.8u_0 + 40$

$u_1 = 0.8(80) + 40$

$u_1 = 104$

$u_2 = 0.8u_1 + 40$

$u_2 = 0.8(104) + 40$

$u_2 = 123.2$

$u_3 = 0.8u_2 + 40$

$u_3 = 0.8(123.2) + 40$

$u_3 = 138.56$ units deep after 3 hours

15C

(4) $u_{n+1} = 1.5u_n + 2k$ $\quad u_1 = 4k$

$u_2 = 1.5(u_1) + 2k$

$u_2 = 1.5(4k) + 2k$

$u_2 = 8k$

$u_3 = 1.5(u_2) + 2k$

$u_3 = 1.5(8k) + 2k$

$u_3 = 14k$

(5) $u_{n+1} = m u_n + c$

$u_1 = m u_0 + c$

$10 = m(5) + c$

$10 = 5m + c$

$10 - c = 5m$

$m = \frac{10 - c}{5}$

(6) $u_{n+1} = m u_n + c$

$u_1 = m u_0 + c$

$5 = 2m + c$ (1)

$u_2 = m u_1 + c$

$23 = 5m + c$ (2)

$18 = 3m$ (2) − (1)

$m = 6$ subs

15C (6) continued

$m = 6$

sub in (1)

$5 = 2(6) + c$

$5 = 12 + c$

$-7 = c$

$c = -7$

(7) $u_1 = a u_0 + b$

$30 = 10a + b$ (1)

$u_2 = a u_1 + b$

$46 = 30a + b$ (2)

$16 = 20a$ (2) − (1)

$\frac{16}{20} = a$

$a = \frac{4}{5}$

sub in (1)

$30 = 10\left(\frac{4}{5}\right) + b$

$30 = 8 + b$

$b = 22$

$u_{n+1} = \frac{4}{5} u_n + 22$

15C ⑦ continued

$u_3 = \frac{4}{5} u_2 + 22$

$u_3 = \frac{4}{5}(46) + 22$

$u_3 = 58.8$

$u_4 = \frac{4}{5}(58.8) + 22$

$u_4 = 69.04$

$u_5 = \frac{4}{5}(69.04) + 22$

$u_5 = 77.232$

⑧ 3.5% per month interest = 100% + 3.5% = 103.5% = 1.035

$u_{n+1} = 1.035 u_n - 400$, $u_0 = £4000$

$u_1 = 1.035 u_0 - 400$

$u_1 = 1.035(4000) - 400$

$u_1 = £3740$ owed on March 1st

$u_2 = 1.035(3740) - 400$

$u_2 = £3470.90$ owed on April 1st

$u_3 = £3192.38$ owed on May 1st

⋮ ⋮

$u_{12} = £203.49$ owed on Feb 1st

15C

(8) continued

€203.49 owed on Feb 1st

1.035 × 203.49 = €210.61 owed on Feb 28th

Last payment = €210.61 on March 1st, the customer was wrong.

151)

(1) (a) $u_{n+1} = 0.3u_n + 6$

$a = 0.3$

$b = 6$

A limit exists as $-1 < 0.3 < 1$

$$L = \frac{b}{1-a}$$

$$L = \frac{6}{1-0.3}$$

$$L = \frac{6}{0.7}$$

$$L = \frac{60}{7} \left(= 8\tfrac{4}{7}\right)$$

(c) $u_{n+1} = -0.4u_n + 1$

$a = -0.4$

$b = 1$

A limit exists as $-1 < 0.4 < 1$

$$L = \frac{b}{1-a}$$

$$L = \frac{1}{1--0.4}$$

$$L = \frac{1}{1.4}$$

$$L = \frac{10}{14}$$

$$L = \frac{5}{7}$$

150

$u_{n+1} = -\frac{3}{2}u_n + 8$

1(f) A limit does not exist as $-\frac{3}{2} < -1$

1(g) $3u_{n+1} = -u_n + 0.6$

$u_{n+1} = -\frac{1}{3}u_n + 0.2$ $\quad a = -\frac{1}{3}$ $\quad b = 0.2$

A limit exists as $-1 < -\frac{1}{3} < 1$

$L = \frac{b}{1-a}$

$L = \frac{0.2}{1-\frac{-1}{3}}$

$L = \frac{0.2}{\frac{4}{3}}$

$L = 0.2 \times \frac{3}{4}$

$L = 0.15$

$L = \frac{3}{20}$

151

A limit exists on $-1 < 0.8 < 1$

Q(2)(a) $u_2 = 0.8(5) + 4$

$u_2 = 8$

$u_3 = 0.8(8) + 4$

$u_3 = 10.4$

$u_4 = 0.8(10.4) + 4$

$u_4 = 12.32$

$u_5 = 0.8(12.32) + 4$

$u_5 = 13.856$

$u_6 = 0.8(13.856) + 4$

$u_6 = 15.0848$

$n = 6$

(2)(b) $L = \frac{b}{1-a}$

$L = \frac{4}{1-0.8}$

$L = \frac{4}{0.2}$

$L = \frac{40}{2}$

$L = 20$

Q(3) $L = \frac{b}{1-a}$ $\quad a = m$ $\quad b = 6$

$20 = \frac{6}{1-m}$

$20(1-m) = 6$

$20 - 20m = 6$

$-20m = -14$

$m = \frac{14}{20}$

$m = \frac{7}{10}$

(4) $a = k \quad b = 2$

$$L_u = \frac{b}{1-a}$$

$$L_u = \frac{2}{1-k}$$

$a = 0.5$

$b = 3$

$$L_v = \frac{b}{1-a}$$

$$L_v = \frac{3}{1-0.5}$$

$$L_v = \frac{3}{0.5}$$

$$L_v = 6$$

Limits are equal

$$\frac{2}{1-k} = 6$$

$$2 = 6(1-k)$$

$$2 = 6 - 6k$$

$$-4 = -6k$$

$$\frac{-4}{-6} = k$$

$$k = \frac{2}{3}$$

151

(5) X-Pest removes 60% so 40% = 0.4 remains

$u_{n+1} = 0.4u_n + 450$ A limit exists as $-1 < 0.4 < 1$

$L = \frac{450}{1-0.4}$

$L = \frac{450}{0.6}$

$L = \frac{4500}{6}$ $\quad 6\overline{)4500} = 750$

$L = 750$

Pest Away removes 75% so 25% = 0.25 remains

$V_{n+1} = 0.25 V_n + 550$ A limit exists as $-1 < 0.25 < 1$

$L = \frac{550}{1-0.25}$

$L = \frac{550}{0.75} \times \frac{4}{4}$

$L = \frac{2200}{3}$ $\quad 3\overline{)2200} = 733\frac{1}{3}$

$L = 733\frac{1}{3}$

In the long term Pest Away is more effective

15D

Q6 16% leaves so 84% remains

$u_{n+1} = 0.84u_n + 25$ $\quad u_0 = 50$

where $a = 0.84$ $\quad b = 25$

$$L = \frac{25}{1-0.84}$$

A limit exists as $-1 < 0.84 < 1$

$$L = \frac{25}{0.16}$$

$$L = \frac{25}{16/100}$$

$$L = 25 \times \frac{100}{16}$$

$$= 25 \times \frac{50}{8}$$

$$= 25 \times \frac{25}{4}$$

$$= \frac{625}{4}$$

$$= 156\tfrac{1}{4}\text{ ml}$$

It is not safe to continue the treatment over time as $156\frac{1}{4} > 150$ so the medicine could be harmful.

15D

7 (a) $u_{n+1} = 0.8u_n + 0.7$ (80% left an 20% removed)

$L = \frac{0.7}{1-0.8}$ A limit exists as $-1 < 0.8 < 1$

$L = \frac{0.7}{0.2}$

$L = \frac{7}{2}$ m

$L = 3.5$m

They will grow to a height of 3.5m over time.

(b) Limit of 2.5m required

$L = \frac{b}{1-a}$

~~$2.5 = \frac{0.7}{2.5(1-a)}$~~

$2.5 = \frac{0.7}{1-a}$

$2.5(1-a) = 0.7$

$2.5 - 2.5a = 0.7$

$-2.5a = -1.8$

$a = \frac{1.8}{2.5} = 0.72$

i.e. 72% remains so 28% will need to be trimmed.

150

(8) $100\% + 2.4\% = 102.4\%$

$u_{n+1} = 1.024 u_n + k \qquad u_0 = 100$

$u_1 = 1.024(100) + k$

$u_1 = £(102.40 + k)$

$u_2 = 1.024(102.40 + k) + k$

$u_2 = £(104.8576 + 2.024k)$

$u_3 = 1.024(104.8576 + 2.024k) + k$

$u_3 = 107.37418 + 3.07256k$

$u_3 = 500$

$500 = 107.37418 + 3.07256k$

$500 - 107.37418 = 3.07256k$

$$\frac{500 - 107.37418}{3.07256} = k$$

$k = £127.78$